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In ΔABC the following relationship holds:

$$\frac{a^2}{\cos^2\frac{A}{2}}+\frac{b^2}{\cos^2\frac{B}{2}}+\frac{c^2}{\cos^2\frac{C}{2}}\geq 12R^2$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$\frac{a^2}{\cos^2\frac{A}{2}}+\frac{b^2}{\cos^2\frac{B}{2}}+\frac{c^2}{\cos^2\frac{C}{2}}=\sum_{cyc}\frac{a^2}{\cos^2\frac{A}{2}}=\sum_{cyc}\frac{a^2}{\frac{s(s-a)}{bc}}=$$

$$=\frac{abc}{s}\sum_{cyc}\frac{a}{s-a}=\frac{abc}{s}\cdot\frac{2(2R-r)}{r}=\frac{4Rrs}{s}\cdot\frac{2(2R-r)}{r}=$$

$$=8R(2R-r)\overset{EULER}{\geq}8R\left(2R-\frac{R}{2}\right)=12R^2$$

Equality holds for $a=b=c$.