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In ΔABC the following relationship holds:

$$\sqrt{(a+b-c)(b+c-a)(c+a-b)} \leq \frac{3\sqrt{3}abc}{(a+b+c)\sqrt{a+b+c}}$$

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Solution by Daniel Sitaru-Romania

$$\sqrt{(a+b-c)(b+c-a)(c+a-b)} \leq \frac{3\sqrt{3}abc}{(a+b+c)\sqrt{a+b+c}}$$

$$\sqrt{(2s-2c)(2s-2a)(2s-2b)} \leq \frac{3\sqrt{3}\cdot 4RF}{2s\sqrt{2s}}$$

$$2\sqrt{2}\cdot 2s\cdot\sqrt{s(s-a)(s-b)(s-c)} \leq 12\sqrt{3}RF$$

$$2\sqrt{2}\cdot 2s\cdot F \leq 12\sqrt{3}RF$$

$$4\sqrt{2}\cdot\sqrt{2}\cdot s \leq 12\sqrt{3}R$$

$$s \leq \frac{12\sqrt{3}R}{8}$$

$$s \leq \frac{3\sqrt{3}R}{2}\ (MITRINOVICI)$$

Equality holds for $a=b=c$.