

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$(b + c - a)(c + a - b)(a + b - c) \leq 2R\sqrt{2Rh_a h_b h_c}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$(b + c - a)(c + a - b)(a + b - c) \leq 2R\sqrt{2Rh_a h_b h_c}$$

$$(2s - 2a)(2s - 2b)(2s - 2c) \leq 2R \sqrt{2R \cdot \frac{2F}{a} \cdot \frac{2F}{b} \cdot \frac{2F}{c}}$$

$$8(s - a)(s - b)(s - c) \leq 2R \sqrt{\frac{4F^2 \cdot 4RF}{abc}}$$

$$8s(s - a)(s - b)(s - c) \leq 2Rs \cdot \sqrt{\frac{4F^2 \cdot 4RF}{4RF}}$$

$$8F^2 \leq 2Rs \cdot 2F, \quad 2F \leq Rs, \quad 2rs \leq Rs$$

$$R \geq 2r \text{ (EULER)}$$

Equality holds for $a = b = c$.