ROMANIAN MATHEMATICAL MAGAZINE

If I −incenter in ΔABC then:

$$IA^2\cdot IB^2\cdot IC^2\le\frac{8}{27}R^3h_ah_bh_c$$

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Solution by Daniel Sitaru-Romania

$$IA^2\cdot IB^2\cdot IC^2\le\frac{8}{27}R^3h_ah_bh_c$$

$$\frac{r^2}{\sin^2\frac{A}{2}}\cdot\frac{r^2}{\sin^2\frac{A}{2}}\cdot\frac{r^2}{\sin^2\frac{A}{2}}\le\frac{8}{27}R^3\cdot\frac{2F}{a}\cdot\frac{2F}{b}\cdot\frac{2F}{c}$$

$$\frac{r^6}{\left(\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)^2}\le\frac{8}{27}\cdot\frac{4RF}{abc}\cdot 2(RF)^2$$

$$\frac{r^6}{\left(\frac{r}{4R}\right)^2}\le\frac{16}{27}\cdot\frac{4RF}{4RF}\cdot(RF)^2$$

$$16R^2r^4\le\frac{16}{27}\cdot R^2\cdot F^2$$

$$27r^4\le F^2,\quad F\ge 3\sqrt{3}r^2,\quad rs\ge 3\sqrt{3}r^2$$

$$s\ge 3\sqrt{3}r\ (MITRINOVICI)$$

Equality holds for $a=b=c$.