

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$a^2 \cos A + b^2 \cos B + c^2 \cos C \leq \frac{9R^2}{2}$$

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WLOG: $a \leq b \leq c \Rightarrow a^2 \leq b^2 \leq c^2$, $\cos A \geq \cos B \geq \cos C$

$$\sum_{cyc} a^2 \cos A \leq \frac{1}{3} \cdot \sum_{cyc} a^2 \cdot \sum_{cyc} \cos A = \frac{2}{3} \cdot (s^2 - r^2 - 4Rr) \cdot \left(1 + \frac{r}{R}\right) \leq$$

$$\stackrel{\text{GERRETSEN}}{\leq} \frac{2}{3} \cdot (4R^2 + 4Rr + 3r^2 - r^2 - 4Rr) \cdot \left(1 + \frac{r}{R}\right) \leq$$

$$\stackrel{\text{EULER}}{\leq} \frac{2}{3} \cdot (4R^2 + 2r^2) \cdot \left(1 + \frac{1}{2}\right) = 4R^2 + 2r^2 \leq$$

$$\stackrel{\text{EULER}}{\leq} 4R^2 + 2 \cdot \frac{R^2}{4} = \frac{9R^2}{2}$$

Equality holds for $a = b = c$.