ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$a\sec\frac{A}{2}+b\sec\frac{B}{2}+c\sec\frac{C}{2}\geq 12r$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$a\leq b\leq c\rightarrow \cos\frac{A}{2}\leq\cos\frac{B}{2}\leq\cos\frac{C}{2}\rightarrow\frac{1}{\cos\frac{A}{2}}\geq\frac{1}{\cos\frac{B}{2}}\geq\frac{1}{\cos\frac{C}{2}}\rightarrow$$

$$\rightarrow\sec\frac{A}{2}\leq\sec\frac{B}{2}\leq\sec\frac{C}{2}$$

$$\sum_{cyc}a\sec\frac{A}{2}\overset{CEBYSHEV}{\overbrace{\geq}}\frac{1}{3}\sum_{cyc}a\cdot\sum_{cyc}\sec\frac{A}{2}\overset{JENSEN}{\overbrace{\leq}}$$

$$\leq\frac{2s}{3}\cdot 3\sec\left(\frac{A+B+C}{6}\right)=2s\cdot\sec\frac{\pi}{6}\overset{MITRINOVIC}{\overbrace{\geq}}$$

$$\geq 2\cdot 3\sqrt{3}r\cdot\frac{1}{\cos\frac{\pi}{6}}=2\cdot 3\sqrt{3}r\cdot\frac{1}{\frac{\sqrt{3}}{2}}=12r$$

Equality holds for: $a=b=c$.