ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$(a+b)m_cw_c+(b+c)m_aw_a+(c+a)m_bw_b \ge 2s(s^2-r^2-4Rr)$$

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$$We\ \ know\ that\ m_c \ge \frac{a+b}{2}\cos\frac{C}{2}$$

$$(a+b)m_cw_c \ge (a+b)\frac{a+b}{2}\cos\frac{C}{2}\cdot\frac{2ab}{a+b}\cos\frac{C}{2}=(a+b)ab\cos^2\frac{C}{2}=$$

$$=(a+b)ab\frac{s(s-c)}{ab}=s(s-c)(a+b)=s(s-c)(2s-c)=s(2s^2-3cs+c^2)$$

$$(a+b)m_cw_c+(b+c)m_aw_a+(c+a)m_bw_b=\sum(a+b)m_cw_c \ge$$

$$\ge s\sum(2s^2-3cs+c^2)=s\left(6s^2-3s(a+b+c)+(a^2+b^2+c^2)\right)=$$

$$=s\left(6s^2-6s^2+2(s^2-r^2-4Rr)\right)=2s(s^2-r^2-4Rr)$$

$$Equality\ holds\ for\ a=b=c$$