ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$h_a + h_b + h_c \geq \sqrt{\frac{2r}{R}}\,(w_a + w_b + w_c)$$

Proposed by Nguyen Minh Tho-Vietnam

Solution by Tapas Das-India

$$\frac{w_a}{h_a} = \frac{2\sqrt{bc.s(s-a)}}{b+c}\frac{2R}{bc} = \frac{4R\sqrt{s(s-a)}}{\sqrt{bc}(2s-a)} = \frac{4R\sqrt{s(s-a)}}{\sqrt{bc}(2(s-a)+a)} \overset{AM-GM}{\leq}$$

$$\leq \frac{4R\sqrt{s(s-a)}}{\sqrt{bc}2\sqrt{2(s-a).a}} = \frac{2R\sqrt{s}}{\sqrt{2abc}} = \frac{2R\sqrt{s}}{\sqrt{8Rrs}} = \sqrt{\frac{R}{2r}}\ (1)$$

$$\sqrt{\frac{2r}{R}}\,(w_a + w_b + w_c) = \sqrt{\frac{2r}{R}}\,w_a + \sqrt{\frac{2r}{R}}\,w_b + \sqrt{\frac{2r}{R}}\,w_c \overset{(1)}{\leq}$$

$$\leq \frac{h_a}{w_a}.w_a + \frac{h_b}{w_b}.w_b + \frac{h_c}{w_c}.w_c = h_a + h_b + h_c$$

$$Equality\ holds\ for\ a = b = c$$