ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{R}{r} \geq 2 + \frac{(a+b+c)((a-b)^2+(b-c)^2+(c-a)^2)}{4abc}$$

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Solution by Tapas Das-India

$$\frac{(a+b+c)((a-b)^2+(b-c)^2+(c-a)^2)}{4abc} = 2s \cdot \frac{2(a^2+b^2+c^2-ab-bc-ca)}{4.4Rrs} =$$

$$= \frac{1}{4Rr}(s^2-3r^2-12Rr) \overset{Gerretsen}{\leq} \frac{1}{4Rr}(4R^2+4Rr+3r^2-3r^2-12Rr) =$$

$$= \frac{1}{4Rr}(4R^2-8Rr) = \frac{R}{r} - 2$$

$$2 + \frac{(a+b+c)((a-b)^2+(b-c)^2+(c-a)^2)}{4abc} \leq 2 + \frac{R}{r} - 2 = \frac{R}{r}$$

Equality holds for an equilateral triangle.