ROMANIAN MATHEMATICAL MAGAZINE

For any triangle ABC prove that :

$$6\left(\tan^2\frac{A}{2}+\tan^2\frac{B}{2}\right)+\tan^2\frac{C}{2}\geq 3$$

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By AM − GM inequality, we have

$$6\left(\tan^2\frac{A}{2}+\tan^2\frac{B}{2}\right)+\tan^2\frac{C}{2}$$
$$=\frac{3}{2}\left(\tan^2\frac{A}{2}+\tan^2\frac{B}{2}\right)+\frac{1}{2}\left(9\tan^2\frac{B}{2}+\tan^2\frac{C}{2}\right)+\frac{1}{2}\left(9\tan^2\frac{A}{2}+\tan^2\frac{C}{2}\right)$$

$$\geq 3\tan\frac{A}{2}\tan\frac{B}{2}+3\tan\frac{B}{2}\tan\frac{C}{2}+3\tan\frac{C}{2}\tan\frac{A}{2}=3.$$

Equality holds iff $A=B=2\tan^{-1}\left(\frac{1}{\sqrt{7}}\right), C=\pi-4\tan^{-1}\left(\frac{1}{\sqrt{7}}\right)$.