ROMANIAN MATHEMATICAL MAGAZINE

In acute ΔABC the following relationship holds:

$$\frac{6}{\pi}<\frac{1}{f(A)}+\frac{1}{f(B)}+\frac{1}{f(C)}<3,\qquad \int_0^{\frac{\pi}{2}}\frac{dx}{1+\cos\alpha\cos x}=f(\alpha)$$

Proposed by Tapas Das-India

Solution by Kartick Chandra Betal-India

$$f(\alpha)=\int_0^{\frac{\pi}{2}}\frac{dx}{1+\cos\alpha\cos x}=2\int_0^{\frac{\pi}{4}}\frac{dx}{1+\cos\alpha\left\{\frac{1-\tan^2 x}{1+\tan^2 x}\right\}}=$$

$$=2\int_0^{\frac{\pi}{4}}\frac{\sec^2 x\,dx}{1+\cos\alpha+(1-\cos\alpha)\tan^2 x}=$$

$$=\frac{2}{\sqrt{1-\cos^2\alpha}}\left(\tan^{-1}\left(\tan\frac{\alpha}{2}\tan x\right)\right)_0^{\frac{\pi}{4}}=\frac{\alpha}{\sin\alpha}$$

$$\frac{1}{f(A)}+\frac{1}{f(B)}+\frac{1}{f(C)}=\frac{\sin A}{A}+\frac{\sin B}{B}+\frac{\sin C}{C}$$

$$\frac{2}{\pi}<\sin x<1\ for\ 0<x<\frac{\pi}{2}$$

$$\frac{6}{\pi}<\frac{1}{f(A)}+\frac{1}{f(B)}+\frac{1}{f(C)}<3$$