# ROMANIAN MATHEMATICAL MAGAZINE


**In acute $\Delta ABC$ the following relationship holds:**

$$a\sqrt{\sin A}+\ b\sqrt{\sin B}+\ c\sqrt{\sin C}>\frac{4}{3}s$$


*Proposed by Vasile Mircea Popa-Romania*

*Solution by Tapas Das-India*


$$We\ know\ that: \cos A\cos B\cos C=\frac{s^2-(2R+r)^2}{4R^2}$$

$$for\ acute\ \Delta ABC\ , \cos A\cos B\cos C>0$$

$$\frac{s^2-(2R+r)^2}{4R^2}>0\ or\ s>(2R+r)\ (1)$$

$$\sin A<1\ then\ \sqrt{\sin A}>\sin A\ \ (2)$$

$$a\sqrt{\sin A}+\ b\sqrt{\sin B}+\ c\sqrt{\sin C}\overset{Chebyshev}{\geq}\frac{1}{3}\left(\sum a\right)\left(\sum\sqrt{\sin A}\right)\overset{(2)}{>}$$

$$>\frac{1}{3}\left(\sum a\right)\left(\sum\sin A\right)=\frac{1}{3}2s\left(\frac{s}{R}\right)>$$

$$\overset{(1)}{>}\frac{1}{3}2s\left(\frac{2R+r}{R}\right)>\frac{2s}{3}.\frac{2R}{R}=\frac{4s}{3}$$