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In ΔABC the following relationship holds:

$$\frac{\cot\frac{A}{2}}{a}+\frac{\cot\frac{B}{2}}{b}+\frac{\cot\frac{C}{2}}{c}\geq\frac{3}{R}$$

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Solution by Daniel Sitaru-Romania

$$\frac{\cot\frac{A}{2}}{a}+\frac{\cot\frac{B}{2}}{b}+\frac{\cot\frac{C}{2}}{c}\overset{AM-GM}{\geq}3\cdot\sqrt[3]{\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}\cdot\frac{1}{abc}}=$$

$$=3\cdot\sqrt[3]{\frac{s}{r}\cdot\frac{1}{4RF}}=3\cdot\sqrt[3]{\frac{s}{4Rr\cdot rs}}=3\cdot\sqrt[3]{\frac{1}{R\cdot(2r)^2}}\geq$$

$$\overset{EULER}{\geq}3\cdot\sqrt[3]{\frac{1}{R\cdot\left(2\cdot\frac{R}{2}\right)^2}}=\frac{3}{R}$$

Equality holds for $a=b=c$.