ROMANIAN MATHEMATICAL MAGAZINE

Find a closed form:

$$\Omega = \int_1^{e^\pi}\int_0^1 \frac{\arcsin(x^a)\arccos(x^a)}{x}\,dxda$$

Proposed by Ankush Kumar Parcha-India

Solution by Ose Favour-Nigeria

$$\Omega = \int_0^1 \frac{\arccos(x)\arcsin(x)}{x}dx = \int_0^1 \frac{\arcsin(x)}{x}\left(\frac{\pi}{2} - \arcsin(x)\right)dx =$$

$$= \frac{\pi}{2}\int_0^1 \frac{\arcsin(x)}{x}dx - \int_0^1 \frac{(\arcsin(x))^2}{x}dx \overset{x=\sin(y)}{\cong}$$

$$= \frac{\pi}{2}\underbrace{\int_0^{\frac{\pi}{2}} yctg(y)dy}_{\ln(2)\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} y^2 ctg(y)dy = \frac{\pi^2}{4}\ln(2) + 2\int_0^{\frac{\pi}{2}} yln(\sin(y))dy =$$

$$= \frac{\pi^2}{4}\ln(2) - 2(\frac{\pi^2}{8}\ln(2) - 2\sum_{k=1}^{\infty}\frac{1}{k}\int_0^{\frac{\pi}{2}} ycos(2ky)dy = \frac{7}{8}\zeta(3)$$