# ROMANIAN MATHEMATICAL MAGAZINE

**Find a closed form:**

$$I=\int_1^{\infty}\frac{Li_2[(1-y)^3]}{(y-1)^3}dy$$

*Proposed by Aryan Desai-India*

*Solution by Quadri Faruk Temitope-Nigeria*

$$I=\int_1^{\infty}\frac{Li_2[(1-y)^3]}{(y-1)^3}dy$$

$$I\overset{y-1=x}{\triangleq}\int_0^{\infty}\frac{Li_2[-x^3]}{x^3}dx,\qquad I\overset{x^3=p}{\triangleq}\int_0^{\infty}\frac{Li_2[-p]}{p}\left(\frac{1}{3}p^{-\frac{2}{3}}\right)dp$$

$$I=\frac{1}{3}\int_0^{\infty}p^{-1-\frac{2}{3}}Li_2(-p)\,dp$$

$$I\overset{Integration\ By\ Parts}{\triangleq}-\frac{1}{2}\int_0^{\infty}p^{-\frac{5}{3}}\ln(1+p)\,dp$$

$$I\overset{Integration\ By\ Parts}{\triangleq}-\frac{3}{2}\int_0^{\infty}\frac{p^{-\frac{2}{3}}}{1+\mathrm{p}}dp$$

$$I=-\frac{3}{2}B\left(\frac{2}{3},\frac{1}{3}\right),\qquad I=-\frac{\frac{3}{2}\Gamma\left(\frac{2}{3}\right)\Gamma\left(\frac{1}{3}\right)}{\Gamma\left(\frac{1}{3}+\frac{2}{3}\right)}$$

$$I=-\frac{3}{2}\Gamma\left(1-\frac{1}{3}\right)\Gamma\left(\frac{1}{3}\right)$$

$$I\overset{Euler\ Reflection}{\triangleq}-\frac{3}{2}\pi\mathrm{cosec}\left(\frac{\pi}{3}\right)=-\frac{\sqrt{3}\pi}{2}$$