ROMANIAN MATHEMATICAL MAGAZINE

Find a closed form:

$$\int_0^{\frac{\pi}{2}} \frac{1}{(\sin(x)+1)^2\sqrt{\ln(tan(x)+sec(x))}}dx$$

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Solution by Rana Ranino-Algeria

$$\Omega = \int_0^{\frac{\pi}{2}} \frac{1}{(\sin(x)+1)^2\sqrt{\ln(tan(x)+sec(x))}}dx \overset{x\to\frac{\pi}{2}-x}{\cong}$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{(\cos(x)+1)^2\sqrt{\ln(ctg(x)+csc(x))}}dx =$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \frac{1}{cos^2\left(\frac{x}{2}\right)\sqrt{-\ln\left(\tan\left(\frac{x}{2}\right)\right)}}\frac{dx}{cos^2\left(\frac{x}{2}\right)} \overset{2x\to x}{\cong}$$

$$= \frac{1}{2}\int_0^{\infty} \frac{e^{-t}\left(1+e^{-2t}\right)}{\sqrt{t}}dt = \frac{1}{2}\int_0^{\infty} t^{\frac{1}{2}-1}e^{-t}\,dt + \frac{1}{2}\int_0^{\infty} t^{\frac{1}{2}-1}e^{-3t}\,dt = \frac{1}{2}\Gamma\left(\frac{1}{2}\right) + \frac{1}{2\sqrt{3}}\Gamma\left(\frac{1}{2}\right) =$$

$$= \frac{\sqrt{\pi}}{2}\left(1+\frac{1}{\sqrt{3}}\right)$$

$$\int_0^{\frac{\pi}{2}} \frac{1}{(\sin(x)+1)^2\sqrt{\ln(tan(x)+sec(x))}}dx = \frac{\sqrt{\pi}}{2}\left(1+\frac{1}{\sqrt{3}}\right)$$