ROMANIAN MATHEMATICAL MAGAZINE

Find a closed form:

$$\Omega=\int_0^1\int_0^1\int_0^1\frac{1}{ln^2(xyz)(1+x^2y^2z^2)}dxdydz$$

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Solution by Obiajunwa Januarius-Nigeria

It is known that:

$$\int_0^1\int_0^1\int_0^1 F(xyz)dxdydz=\frac{1}{2}\int_0^1 ln^2(x)F(x)\,dx$$

$$\Omega=\int_0^1\int_0^1\int_0^1\frac{1}{ln^2(xyz)(1+x^2y^2z^2)}dxdydz=\frac{1}{2}\int_0^1\frac{ln^2(x)}{(1+x^2)ln^2(x)}dx=\frac{1}{2}\int_0^1\frac{dx}{1+x^2}=\frac{\pi}{8}$$