ROMANIAN MATHEMATICAL MAGAZINE

Find a closed form:

$$I=\int_0^{\infty}\frac{(z^2+1)z^{106}\ln(z)}{(z^{100}+1)^2(z^4+\frac{2}{5}z^2+1)^4}dz$$

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Solution by Pham Duc Nam-Vietnam

$$I=\int_0^{\infty}\frac{(z^2+1)z^{106}\ln(z)}{(z^{100}+1)^2(z^4+\frac{2}{5}z^2+1)^4}dz=625\int_0^{\infty}\frac{(z^2+1)z^{106}\ln(z)}{(z^{100}+1)^2(5z^4+2z^2+5)^4}dz=$$

$$625\left(\int_0^{1}\frac{(z^2+1)z^{106}\ln(z)}{(z^{100}+1)^2(5z^4+2z^2+5)^4}dz+\int_1^{\infty}\frac{(z^2+1)z^{106}\ln(z)}{(z^{100}+1)^2(5z^4+2z^2+5)^4}dz\right)$$

$$\textit{Change the variable } z=\frac{1}{t} \textit{ for the latter integral yields}:$$

$$I=625\int_0^{1}\frac{(z^2+1)z^{106}\ln(z)}{(z^{100}+1)^2(5z^4+2z^2+5)^4}dz+$$

$$625\int_1^{\infty}\frac{\left((\frac{1}{t})^2+1\right)(\frac{1}{t})^{106}\ln\left(\frac{1}{t}\right)}{t^2\left((\frac{1}{t})^{100}+1\right)^2\left(5(\frac{1}{t})^4+2(\frac{1}{t})^2+5\right)^4}dt$$

$$I=625(\int_0^{1}\frac{(z^2+1)z^{106}\ln(z)}{(z^{100}+1)^2(5z^4+2z^2+5)^4}dz-\int_0^{1}\frac{(t^2+1)t^{106}\ln(t)}{(t^{100}+1)^2(5t^4+2t^2+5)^4}dt=0$$