ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\int_1^2 \frac{dx}{x\sqrt{1+x^3}}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Mirsadix Muzefferov-Azerbaijan

$$\int_1^2 \frac{dx}{x\sqrt{1+x^3}} = \int_1^2 \frac{x^2dx}{x^3\sqrt{1+x^3}} = \frac{1}{3}\int_1^2 \frac{d(1+x^3)}{x^3\sqrt{1+x^3}} \overset{\sqrt{1+x^3}\to t}{\triangleq}$$

$$= \frac{1}{3}\int_{\sqrt{2}}^3 \frac{dt^2}{(t^2-1)t} = \frac{2}{3}\int_{\sqrt{2}}^3 \frac{tdt}{t(t^2-1)} = -\frac{2}{3}\int_{\sqrt{2}}^3 \frac{dt}{(1-t^2)} =$$

$$= -\frac{2}{3}ln\left|\frac{1+t}{1-t}\right|\Bigg|_{\sqrt{2}}^{3} = -\frac{2}{3}\left(\ln(2) - \ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)\right) =$$

$$= -\frac{2}{3}\left(\ln(2) - \ln\left(\sqrt{2}+1\right)^2\right) = \frac{2}{3}\left(\ln\left(3+2\sqrt{2}\right) - \ln(2)\right) =$$

$$= \frac{2}{3}\ln\left(\frac{3+2\sqrt{2}}{2}\right) = \frac{2}{3}\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right) = \frac{2}{3}\ln\left(1+\frac{1}{\sqrt{2}}\right)$$