ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$I = \int_1^{\infty} \frac{x}{\sqrt{x^4 + x^2 + 1}} dx$$

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$$I = \int_1^{\infty} \frac{x}{\sqrt{x^4 + x^2 + 1}} dx = \int_1^{\infty} \frac{dx}{\sqrt{x^2 + \frac{1}{x^2} + 1}} =$$

$$x = \frac{1}{y} \to dx = -\frac{1}{y^2} dy \to = \int_1^{\infty} \frac{-\frac{1}{y^2} dy}{\sqrt{y^2 + \frac{1}{y^2} + 1}}$$

$$I = \int_1^{\infty} \frac{dx}{\sqrt{x^2 + \frac{1}{x^2} + 1}}, I = \int_1^{\infty} \frac{-\frac{1}{x^2} dx}{\sqrt{x^2 + \frac{1}{x^2} + 1}} \Rightarrow 2I = \int_1^{\infty} \frac{(1 - \frac{1}{x^2}) dx}{\sqrt{x^2 + \frac{1}{x^2} + 1}}$$

$$x + \frac{1}{x} = y \to \left(x + \frac{1}{x}\right)' dx = (1 - \frac{1}{x^2}) dx = dy$$

$$2I = \int_1^{\infty} \frac{\left(x + \frac{1}{x}\right)' dx}{\sqrt{\left(x + \frac{1}{x}\right)^2 - 1}} = \int_2^{\infty} \frac{dy}{\sqrt{y^2 - 1}} = log \left| y + \sqrt{y^2 - 1} \right| \Big|_2^{\infty} = \infty$$