ROMANIAN MATHEMATICAL MAGAZINE

Find a closed form:

$$\Omega=\int_1^\infty \frac{x^2}{x^4+x^2+1}dx$$

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$$\Omega=\int_1^\infty \frac{x^2}{x^4+x^2+1}dx=\int_1^\infty \frac{x^2}{x^2\left(x^2+\frac{1}{x^2}+1\right)}dx=\int_1^\infty \frac{1}{x^2+\frac{1}{x^2}+1}dx=$$

$$\overset{x=\frac{1}{y}}{=}\int_1^\infty \frac{-\frac{1}{y^2}}{y^2+\frac{1}{y^2}+1}dy=\int_1^\infty \frac{-\frac{1}{x^2}}{x^2+\frac{1}{x^2}+1}dx$$

$$2\Omega=\int_1^\infty \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}+1}dx$$

$$\Omega=\frac{1}{2}\int_1^\infty \frac{\left(x+\frac{1}{x}\right)'dx}{\left(x+\frac{1}{x}\right)^2-3}\overset{y=x+\frac{1}{x}}{=}\frac{1}{2}\int_2^\infty \frac{1}{y^2-3}dy=$$

$$\Omega=\frac{1}{4\sqrt{3}}\lim_{y\to\infty}\left(ln\left|\frac{y-\sqrt{3}}{y+\sqrt{3}}\right|\right)-\frac{1}{4\sqrt{3}}ln\left|\frac{2-\sqrt{3}}{2+\sqrt{3}}\right|$$

$$\Omega=\frac{1}{4\sqrt{3}}ln1-\frac{1}{4\sqrt{3}}ln\left|\frac{2-\sqrt{3}}{2+\sqrt{3}}\right|=\frac{1}{4\sqrt{3}}ln\left(\frac{2+\sqrt{3}}{2-\sqrt{3}}\right)$$