ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\int_0^{\frac{\pi}{2}} \frac{\sin(2x)+\sin(x)}{\sqrt{1+5\cos(x)}}\,dx$$

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$$\Omega = \int_0^{\frac{\pi}{2}} \frac{\sin(2x)+\sin(x)}{\sqrt{1+5\cos(x)}}\,dx = \int_0^{\frac{\pi}{2}} \frac{\sin(x)\,(1+2\cos(x))}{\sqrt{1+5\cos(x)}}\,dx =$$

$$-\int_0^{\frac{\pi}{2}} \frac{(1+2\cos(x))}{\sqrt{1+5\cos(x)}}\,d(\cos(x)) \overset{\cos(x)\to t}{\triangleq} -\frac{1}{5}\int_1^0 \frac{(1+2t)}{\sqrt{1+5t}}\,d((1+5t) =$$

$$-\frac{1}{5}\int_1^0 (1+2t)d(2\sqrt{1+5t}) = -\frac{2}{5}\int_1^0 (1+2t)d(\sqrt{1+5t}) \overset{\sqrt{1+5t}\to u}{\triangleq}$$

$$\frac{2}{5}\int_1^{\sqrt{6}} \frac{(2u^2+3)}{5}\,du = \frac{2}{25}\int_1^{\sqrt{6}} (2u^2+3)\,du$$

$$\left\{1+5t=u^2 \to\ t=\frac{u^2-1}{5} \to\ 1+2t=1+\frac{2u^2-2}{5}=\frac{2u^2+3}{5}\right\}$$

$$\frac{2}{25}\int_1^{\sqrt{6}} (2u^2+3)\,du = \frac{2}{25}\left(\frac{2}{3}u^3+3u\right)\Bigg|_1^{\sqrt{6}} = \frac{2}{25}\left(7\sqrt{6}-\frac{11}{3}\right)$$