ROMANIAN MATHEMATICAL MAGAZINE

Find a closed form:

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{\cos(x)-\sin(x)}\,dx$$

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Solution by Odeyemi Gideon-Nigeria

$$I=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{\cos(x)-\sin(x)}\,dx=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)}\,dx$$

$$I=\sqrt{\sqrt{2}}\int_{0}^{\frac{\pi}{2}} \sqrt{\cos(x)}dx=\sqrt{\sqrt{2}}\int_{0}^{\frac{\pi}{2}} cos^{2\left(\frac{3}{4}\right)-1}(x)sin^{2\left(\frac{1}{2}\right)-1}(x)dx$$

$$I=\sqrt{\sqrt{2}}\frac{\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{1}{2}\right)}{2\Gamma\left(\frac{3}{4}+\frac{1}{2}\right)}=\sqrt{\sqrt{2}}\frac{\sqrt{\pi}\Gamma\left(\frac{3}{4}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{\cos(x)-\sin(x)}\,dx=\sqrt{\pi\sqrt{2}}\frac{\Gamma\left(\frac{3}{4}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$